THE EXPECTED DURATION OF GAMMA-RAY BURSTS IN THE IMPULSIVE HYDRODYNAMIC MODELS

Abhas Mitra[1]

Received ______________________; accepted ______________________

[1]Theoretical Physics Division, Bhabha Atomic Research Center, Mumbai- 400 085, India

E-mail: nrl@magnum.barc.ernet.in

ABSTRACT

Depending upon the various models and assumptions, the existing literature on Gamma Ray Bursts (GRBs) mentions that the gross theoretical value of the duration of the burst in the hydrodynamical models is $\tau \sim r_\gamma/2\eta^2 c$, where r_γ is the radius at which the blastwave associated with the fireball (FB) becomes radiative and sufficiently strong. Here $\eta \equiv E/Mc^2$, c is the speed of light, E is initial lab frame energy of the FB, and M is the baryonic mass of the same (Rees & Meszaros 1992). However, within the same basic framework, some authors (like Katz and Piran) have given $\tau \sim r_\gamma/2\eta c$. We intend to remove this confusion by considering this problem at a level deeper than what has been considered so far. Our analysis shows that none of the previously quoted expressions are exactly correct and in case the FB is produced impulsively and the radiative processes responsible for the generation of the GRB are sufficiently fast, its expected duration would be $\tau \sim a r_\gamma/2\eta^2 c$, where $a \sim O(10^1)$. We further discuss the probable change, if any, of this expression, in case the FB propagates in an anisotropic fashion. We also discuss some associated points in the context of the Meszaros & Rees scenario.

Subject headings: gamma rays: bursts- hydrodynamics-relativity

1. INTRODUCTION

Our present understanding of the phenomenon of GRBs is based on the foundations laid by Cavallo & Rees (1978), Goodman (1986), Paczynski (1986), Eichler et al. (1989), Shemi & Piran (1990) and several other works. However, as far as the origin of actually observed nonthermal and highly complex spectra are concerned, we are indebted to another important idea (Rees & Meszaros 1992, Meszaros & Rees 1993) that cosmic FBs with appreciable pollution of baryonic mass, i.e., with a value of $\eta \sim 10^2 - 10^4$, where η has been defined in the abstract, could be a virtue rather than being a problem. Most of our current efforts to understand the phenomenology of GRBs largely hinge on this above mentioned framework. Meszaros & Rees (1993) suggested that the duration of the burst should be $\tau \sim r_{\rm d}/2\eta^2 c$, where $r_{\rm d}$ is the so-called deceleration radius measured in the lab frame. At $r = r_{\rm d}$, the FB is supposed to transmit half of its original momentum to the medium. The baryon polluted FB is expected to drive a strong forward shock (or a blast wave) in the ambient medium, presumably the interstellar medium (ISM), and at $r = r_{\rm d}$, the blast wave is assumed to be sufficiently strong as the FB transfers half of its momentum to it. Technically, we can define another distance, namely, r_γ, where either the blast wave or the reverse shock becomes sufficiently radiative as far as hard X-ray and gamma ray (i.e., the main component of the observed GRBs) productions are concerned. The value of r_γ will be highly model dependent and contains all the microphysics of the process, and strictly speaking, there may not be any simple correlation between the values of $r_{\rm d}$ (largely a simpler hydrodynamic definition) and r_γ (a highly model dependent definition). We want to emphasize here that the basic definition of τ should naturally involve r_γ and not $r_{\rm d}$. Unfortunately, the present status of the studies on GRB is quite preliminary, and, it is not possible to unambiguosly define the value of r_γ even for a relatively simple model. In this situation, practically, all the authors tacitly assume that $r_\gamma \sim r_{\rm d}$. Since the present paper endeavours to analyze the question of the gross time scale in the context of the existing framework of hydrodynamical model of GRBs we will also use the condition $r_\gamma \sim r_{\rm d}$ although we will try to retain the physical distinction between r_γ and $r_{\rm d}$ as far as possible.

However, following the same basic framework, Katz (1994, see his eqn. 23) finds that for a FB propagating in a dense ambient medium (which could be a molecular cloud), we should have $\tau \sim r_{\rm d}/\eta c$. Similarly, Piran (1994) also concludes that $\tau \sim r_{\rm d}/\eta c$ (see his eqn. 25). Although, in a subsequent paper (Sari & Piran 1995), the question of hydrodynamical timescales and temporal structure of GRBs has been discussed with considerable detail, we feel that the aspect of the gross overall duration has not been answered in an umambiguous manner. This foregoing work, in particular, laid specific emphasis on the temporal structure of the GRBs associated with the occurrence of the Newtonian, or, subsequently, relativistic, reverse shock which is supposed to propagate inside the FB. On the other hand, we would discuss that, if the fireball (FB) actually becomes radiative at the deceleration radius (where it loses half of its initial momentum and kinetic energy to the ambient medium), the likely evolution of the reverse shock from a Newtonian one to a fully relativistic one (at $r \gg r_{\rm d}$) could be of somewhat academic interest. However, in case the blastwave fails to become radiative at $r \sim r_{\rm d}$ for some reason or another, the considerations due to Sari & Piran (1995) may become applicable to the actual cases. Here again, one has to address the question of the overall gross duration of the bursts.

To fully appreciate the origin of this dilemma and its resolution, we would revisit the concept of duration of pulses emitted by a FB propagating in a vacuum. For the sake of clarity, we would use the following nomenclature: $\gamma \rightarrow$ the instantaneous bulk Lorentz factor (LF) of *any* fluid emitting the radiation and $\gamma_{\rm F} \rightarrow$ the instantaneous bulk LF of the FB. Thus γ is a general nomenclature applicable to both the original FB (in a vacuum) or any section of the shocked fluid (in a medium) which might be emitting radiation whereas $\gamma_{\rm F}$ specifically refers to the FB.

Note that, the shock can not be sufficiently strong unless $r > r_{\rm d}$. We would see that, in case the GRB eventually results from a shock which becomes sufficiently strong as well as radiative at $r = r_{\rm d} = r_\gamma$, as envisaged by Meszaros & Rees, we should have $\tau \sim a r_{\rm d}/2\eta^2 c$, where $a \sim 10$, which means that in the Meszaros & Rees scheme, the actual duration of the bursts should be at least one order higher than what has been contemplated so far (for a fixed η). We would also try to point out that as long as the blast wave can be considered

sufficiently radiative, for estimating the eventual duration of the burst, the reverse shock plays an insignificant role.

2. FIREBALL IN A VACUUM

When a FB propagates in a vacuum, we have $\gamma \equiv \gamma_{\rm F}$. It is interesting to compare the radiation emitted by the spherically expanding FB with that of a relativistically moving point source showing apparent superluminal (transverse) motion (Rees 1966):

$$v_{\perp} = \frac{v \sin\theta}{1 - \beta \cos\theta} \tag{1}$$

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and θ is the angle between the line of sight and the direction of motion. We may obtain $v_{\perp} > c$ in some circumstances essentially because the source of radiation moves with a speed $v \to c$, and tends to catch up with the radiation emitted by itself. This phenomenon occurs because the velocity of propagation of the radiation from the fast moving source remains fixed at c and does not increase in a Galilean fashion. The same relativistic phenomenon is actually responsible for offering a value of τ much shorter than the expected non-relativistic value $\sim r_\gamma/c$. Nevertheless, for considering the (apparent) time seen by the observer, we have to consider the line of sight velocity of the FB rather than the tranverse velocity:

$$v_{\|} = \frac{v \cos\theta}{1 - \beta \cos\theta} \tag{2}$$

For the exact point of the FB intersected by the line of sight, i.e. for $\theta = 0$, one can see that $v_{\|} = v/(1-\beta) \approx 2\ v\ \gamma_{\rm F}^2$ for $\gamma_{\rm F} >> 1$. Hence $\tau \sim r/v_{\|} \sim r/2c\gamma_{\rm F}^2$. However, the observer receives light not only from a given point but also from other parts of the FB. Because of relativistic aberration, the angular extent of the region is limited to $\theta \sim 1/\gamma_{\rm F}$ in the lab frame. Then we have

$$\tau \sim \frac{r}{c}\frac{1-\beta\cos\theta}{\cos\theta} \sim \frac{r}{c\,\gamma_{\rm F}^2} \tag{3}$$

Here it is assumed that $r = r_\gamma \gg r_0$, the initial radius of the FB. Initially the FB could be non-relativistic and in case it could manange to send radiation outside (actually it would be extremely optically thick), the duration of the burst would have been $\tau_0 \sim r_0/c$. If the explosive energy is liberated not instantaneously, but over a tiny but finite time-scale, the impulsive approximation is correct as long as the expected $\tau \gg \tau_0$. If the FB really becomes optically thin, i.e., the Thomson scattering optical depth of the FB fluid becomes ≤ 1 at $r = r_{\rm T}$, most of the radiation escapes out over a distance $r \sim r_{\rm T}$, and we would have $\tau \sim r/c\gamma_{\rm T}^2$, where $\gamma_{\rm T}$ is the bulk Lorentz factor of the FB at this point. Dynamics of the FB shows that from the initial non-relativistic phase, the FB becomes quickly relativistic and $\gamma_{\rm F} \propto r$, and $\gamma_{\rm F} \rightarrow (\eta+1) \approx \eta$ when $r_\eta = 2\ r_0\ \eta$, and then the FB coasts freely with $\gamma_{\rm F} \approx \eta$ (Piran, Shemi & Narayanan 1993, Meszaros, Laguna, & Rees 1993). For a wide range of parameters of cosmic FBs, it also follows that $r_{\rm T} >> r_\eta$, so that $\gamma_{\rm T} \approx \eta$. At the same time, the observed duration of the burst cannot be smaller than τ_0. Therefore, for the sake of consistency, we will have $\tau = r_{\rm T}/c\eta^2$ if $r_{\rm T} \geq r_0\eta^2$ and $\tau = r_0/c$ otherwise.

This exercise suggests that, in the lab frame, the FB appears as a narrow shell of width $\Delta r \sim r_{\rm T}/{\gamma_{\rm F}}^2$ (if $r_{\rm T} > r_0\eta^2$). The Doppler factor associated with the superluminal motion is $\delta = [\gamma(1-\beta\cos\theta)]^{-1}$ and the the comoving duration of the pulse is (see eq. 2)

$$\tau_c \approx \tau\delta \approx \frac{r}{v}\frac{1}{\gamma_{\rm F}\cos\theta} \approx \frac{r}{c\gamma_{\rm F}} \tag{4}$$

Of course, this result could have been obtained directly by considering the Lorentz contraction of the length r in the fluid frame. For the baryon polluted FBs with $\eta < 10^5 E_{51} r_{0,{\rm F}}^{-2/3}$, the escape of radiation at $r \approx r_{\rm T}$ is only of pedagogic importance because only an insignificant fraction of the FB energy $\sim E/\eta$ is available in the form of FB radiation at $r \approx r_\eta < r_{\rm T}$. Almost the entire energy of the FB gets channelized into the bulk kinetic energy of the baryons and which cannot radiate efficiently in vacuum (Shemi & Piran 1990).

3. FIREBALL IN A MEDIUM

Meszaros & Rees (1993) pointed out that the baryonic FB should sweep the ambient ISM and drive a strong forward shock. In principle, this blast wave may be radiating all the time, but the rate of radiation cannot be substantial unless appreciable amount of the FB momentum and energy have been transmitted to the medium and the shock becomes radiative at $r = r_\gamma$. It is also likely that the FB radiates considerable amount of energy in non-thermal gamma rays even before $r = r_\gamma$ owing to the existence of internal shocks (Rees & Meszaros 1994). However, we implicitly assume here that the fraction of initial energy lost in this way would be $< 50\%$.

We can simplify the BF-shock configuration by a one dimensional sketch following Katz (1994) and Piran (1994) (see Fig. 1). The region (4) represents the unperturbed FB whose original edge S is the contact discontinuity between the piston driving the shock and the unperturbed ISM (1). The region (2) is the perturbed and squeezed-shocked ISM whereas region (3) is the perturbed and reverse-shocked FB. Let $w = e + p$ denote the proper enthalpy density for each region (with appropriate subscript) where e denotes the proper internal energy density and p represents the pressure. Again, some clarifications about the nomenclatures would be in order here. A subscript to the quantities such as e, p and w will represent respective proper values in a given region, i.e. e_1 will represent internal energy density of region 1. In contrast, since, LFs are always meaningful with respect to a certain inertial frame, γ_{12} will represent the value of LF of the region 2 with respect to the region 1 and *vice-versa*. Since, now *the radiation is emitted not by the FB but by the shocked fluid in the region (2) and (3) with a lab frame bulk LF of* $\gamma_{12} = \gamma_{31} = \gamma$ (the lab frame being the ISM at rest [1]) *it is the value of* γ_{12} *rather than* γ_{F} *which now determines the value of* τ. And it is here that the value of τ *bifurcates between* $r_\gamma/c\eta$ *and* $r_\gamma/c\eta^2$ *in the literature*. Therefore, we must unambiguously find the value of γ_{12} in terms of η to settle this issue. Here it may also be emphasized that at least for the 1-D simplification employed by us the lab frame LF of the reverse-shocked fluid in region (3) is also γ_{12} though the LF of it with respect to the region (4) is γ_{34} could be very much different from γ_{31}.

Now we can recall the strong shock jump conditions from Taub (1949) and Blandford & Mckee (1976) and apply the same at S_1, the forward shock:

$$\frac{e_2}{n_2} = \gamma_{12}\frac{w_1}{n_1}, \tag{5}$$

$$\frac{n_2}{n_1} = \frac{\gamma_{12}\Gamma_2 + 1}{\Gamma_2 - 1}, \tag{6}$$

and,

$${\gamma_{S_1}}^2 = \frac{(\gamma_{12}+1)[\Gamma_2(\gamma_{12}-1)+1]^2}{\Gamma_2(2-\Gamma_2)(\gamma_{12}-1)+2} \tag{7}$$

where γ_{S_1} is the LF of the interface S_1 between 1 and 2. Here Γ is defined by the relation

$$p \equiv (\Gamma - 1)(e - \rho) \tag{8}$$

where ρ is the rest mass density in the respective regions (Blandford & Mckee 1976). For a simple one-component fluid, physically Γ is just the ratio of specific heats, and has a value lying between $\frac{4}{3}$ and $\frac{5}{3}$. We expect the (forward) shocked fluid to be highly compressed, heated and to be highly relativistic (internal energy wise), so that, $\Gamma_2 = 4/3$ and $e_2 = p_2/3$. On the other hand, the region (1), i.e., the unperturbed ambient medium is assumed to be cold so that $p_1 \approx 0$ and $w_1 \approx e_1 \approx mn_1c^2$, where m stands for proton mass. Then it promptly follows that

$$e_2 \approx m\, n_2\, c^2\, \gamma_{12}, \tag{9}$$

$$n_2 \approx 4\, \gamma_{12}\, n_1, \tag{10}$$

and,

$$\gamma_{S_1} \approx \sqrt{2}\, \gamma_{12}. \tag{11}$$

If we assume the surface of contact discontinuity to be at perfect pressure equilibrium then we will have $p_3 = p_2$, and, further, assuming the reverse-shocked fluid also to be highly relativistic (as far as internal energy is concerned), i.e., $p_3 = e_3/3$, we find $e_3 \approx e_2$. As to the region (4), i.e., the unshocked part of the FB, the baryons are assumed to be coasting freely after $r > r_\eta << r_\gamma$ with a LF $\gamma_{\rm F} \rightarrow \eta + 1 \approx \eta$ until they sacrifice considerable portion (half at $r = r_{\rm d}$) of their bulk energy to the shocked naterial. Therefore, at $r > r_\eta$, the FB material is also non-relativistic in its rest frame enabling us to write $p_4 \approx 0$, $w_4 \approx e_4 \approx mn_4c^2$. This allows us to form another set of simplified jump conditions at S_2 :

$$\frac{e_3}{n_3} \approx \gamma_{34} \, \frac{w_4}{n_4} = \gamma_{24} \, mc^2 \tag{12}$$

and

$$\frac{n_3}{n_4} \approx 4\gamma_{34} + 3 \approx 4\alpha \, \gamma_{24}, \tag{13}$$

where

$$\alpha \equiv \left(1 + \frac{3}{\gamma_{34}}\right) \tag{14}$$

and the LF of the interface between 3 and 4

$$\gamma_{S_2} \approx \sqrt{2} \, \gamma_{24}. \tag{15}$$

The value of α lies between 1 (ultrarelativistic reverse flow, $\gamma_{34} \gg 1$) and 4 (mildly relativistic reverse flow, $\gamma_{34} \approx 1$). Following Piran (1994), if we define $f \equiv n_4/n_1$, and utilize the fact that $\gamma_{\rm F} = \gamma_{12} \times \gamma_{24}$, we can eliminate γ_{24} from the foregoing equations to obtain

$$\gamma_{12} \approx f^{1/4} \, \gamma_{\rm F}^{1/2} \, \alpha^{1/2}, \tag{16}$$

$$\gamma_{24} = \gamma_{34} \approx f^{-1/4} \, \gamma_{\rm F}^{1/2} \, \alpha^{-1/2}, \tag{17}$$

and

$$n_3 \approx 4 \, f^{-1/4} \, {\gamma_{\rm F}}^{1/2} \, \alpha^{1/2}. \tag{18}$$

Katz (1994) has considered a case with $n_4 = n_1$, i.e., $f = 1$, and $\alpha = 1$ to obtain $\gamma_{12} \approx {\gamma_{\rm F}}^{1/2}$. And thus he obtained $\tau \approx r_\gamma/c\ \gamma_{12}^2 \approx r_\gamma/c\ \gamma_{\rm F}$. Let us now try to see whether this consideration of $f = 1$ is justified or not. As is well understood, each observer sees the FB within a solid angle of $\gamma_{\rm F}^{-2}$, and a spherical FB will appear as a collection of $\gamma_{\rm F}^2$ incoherent beamed FBs to as many observers distributed over 4π solid angle. Thus even for a FB with angular extent of 4π, we can actually take care of considerable amount of anisotropic development. As long as a given ambient medium, which could be the background ISM ($n_1 \approx 1$) or a molecular cloud ($n_1 \sim 10^{2-4}$), has a linear width much larger than the length scales associated with the development and completion of the radiative processes associated with a GRB event we can crudely consider the ambient medium to be uniform over a certain scale. Since the value of r_γ is expected to be $< 10^{17}$ cm, and the associated solid angle, $\gamma_{\rm F}^{-2}$, is expected to be 10^{-4} to 10^{-8} steradian corresponding to $\gamma_{\rm F} \sim 10^2 - 10^4$, the linear scales in question are indeed much smaller than the typical cloud dimensions $\sim$ few pc, and thus, we should not be concerned with the probable different values of n_1. Given a certain fixed value of n_1, we can now uniquely find the value of n_4 :

$$n_4 = \frac{M}{4\pi r^2 (\Delta r)_{\rm com}\ m}. \tag{19}$$

where the comoving width of the FB is $(\Delta r)_{\rm com} = r/\gamma_{\rm F}$. And since $M \approx E/\eta c^2$, we obtain

$$n_4 = \frac{E(\gamma_{\rm F}/\eta)}{4\pi r^3 c^2 m} \approx 5 \times 10^7\ E_{51}\ r_{15}^{-3}\ (\gamma_{\rm F}/\eta)\ {\rm cm}^{-3}. \tag{20}$$

Now, we will have to confront the question: physically, which is the most appropriate definition for r_γ? If the blast wave becomes radiative at $r = r_{\rm d}$, obviously, following Meszaros and Rees (1993), the radiative radius r_γ should be the deceleration radius, $r_{\rm d}$. Simple energy and momentum conservation consideration shows that at the deceleration radius, i.e., the radius when half of the initial momentum gets transmitted, we have $\gamma_{\rm F} = \gamma_{\rm Fd} \approx \eta/2$ and the swept up mass $\approx M/\gamma_{\rm F}$. This yields

$$r_{\rm d} = \left(\frac{3E}{4\pi c^2 \gamma_{\rm Fd} \eta m n_1}\right)^{1/3} \approx \left(\frac{3E}{2\pi c^2 \eta^2 m n_1}\right)^{1/3} \approx 7 \times 10^{15} E_{51}^{1/3} \eta_3^{-2/3} n_1^{-2/3} {\rm cm} \qquad (21)$$

where $\eta_3 = \eta/10^3$ and $n_1 \to n_1/1\ {\rm cm}^{-3}$. We can also contemplate as to which is the fundamental scale length (apart from r_0) in this problem. Could it be the "Sedov length" $l \equiv (E/n_1 m c^2)$ where the FB sweeps a mass equal to E/mc^2 (Sari & Piran 1995)? From a dynamical point of view the concept of momentum exchange is more meaningful than the concept of swept up mass and in a nonrelativistic SNR case, the equality of swept up mass implies equal momentum sharing. Therefore in a relativistic dynamical problem (GRB), it is the "deceleration length" which the physical equivalent to the idea of Sedov Length appearing in the supernova remnant (SNR) theory. As we allow $\gamma_{\rm F} \to 1$, we find, $r_{\rm d} \to l$ within a small numerical factor! If the Sedov Length were indeed a basic length scale in this problem, there would be a basic time-scale $l/c \sim 1$ yr - a time scale which is actually appropriate for the SNR case and which may also be appropriate for low wavelength afterglow following the main GRB.

Of course, it is probable that the shock actually may not be sufficiently radiative at $r = r_{\rm d}$, but it becomes so at a much later distance where the value of f would be much lower than $f_{\rm d}$ (in a spherically symmetric 3-D geometry) and the FB has swept an amount of mass much larger than $M/\gamma_{\rm F}$ because of a variety of reasons (Sari & Piran 1995). Unless we have a specific prescription to describe the extent to which the FB is radiative, the problem becomes rather poorly defined in this case. The most important parameter describing the radiative maturity of the shock could be the *in-situ* magnetic field near the blast wave, and, almost for any model of enhanced magnetic field generation, the value of the magnetic field decreases at least linearly until the background ISM value is achieved (Meszaros, Rees, and & Papathanssiou 1994). Therefore, *it is highly unlikely that if the blast wave fails to become radiative at $r = r_{\rm d}$, it would be radiative at a much larger radius in the framework of a purely hydrodynamic model.* Nevertheless, we would like point out here that this whole discussion explores the question of duration of the GRB in the idealistic Meszaros & Rees framework which assumes that even in a sparse ISM ($n_1 \leq 1$), the hydrodynamic limit

is achieved at any radius. We have discussed elsewhere (Mitra 1996) that actually this framework may not be be valid at all when applied to a sparse ambient medium because because the mean free path of the leading particles of the blast wave (λ) is unlikely to satisfy the condition $\lambda \ll \Delta r$, where $\Delta r \sim r/\gamma^2 c$ is the lab frame width of the FB. This would mean that the hydrodynamical limit may not at all be achieved at the expected value of $r \sim r_{\rm d}$ and the FB may not transfer any appreciable amount of energy and momentum to the ambient medium. Naturally, there may be no strong shock at all at $r = r_{\rm d}$ (Mitra 1996). In such a case, the FB may interact with the ambient medium in the fluid limit at a much larger distance $r \gg r_{\rm d}$ and part of the discussion by Sari & Piran (1995) may be applicable in a surprisingly unexpected way.

For the time being we ignore such disturbing possibility and note that in any case, it is really not necessary for the emission of the gamma-rays that the reverse shock crosses the FB; the blast wave may be radiative enough on its own though it may look for seed photons originating from any source including the reverse shock. Then at $r = r_\gamma = r_{\rm d}$, the foregoing equations lead to

$$f = \frac{n_4}{n_1} = \frac{1}{6}\gamma_{\rm Fd}\ \eta \approx \frac{1}{3}\gamma_{\rm Fd}^2 \approx \frac{1}{12}\eta^2 \tag{22}$$

Therefore, we must have $f >> 1$ at the deceleration radius in the Meszaros and Rees scenario. Now we can go back to eqn. (17 - 22) to find that for $r = r_{\rm d}$, we have

$$\gamma_{12} \approx (1/3)^{1/4}\ \alpha^{1/2}\ \gamma_{\rm Fd} \tag{23}$$

and

$$\gamma_{24} \approx 3^{1/4}\ \alpha^{-1/2} \approx 1. \tag{24}$$

Because of the uncertainty in the value of the bulk LF of the reverse shock material (which we clearly find to be ~ 1) we are still not able to exactly fix the value of γ_{12}. However, we can do so by appealing to some simple physical facts. The first condition is a trivial one that we cannot have the value of any LF, in particular, $\gamma_{24} < 1$. And in order

that there is a forward shock at all, we must have $\gamma_{S_1}(\approx \sqrt{2}\gamma_{12}) \geq \gamma_{\rm F}$. Finally, there will be no reverse shock if $\gamma_{12} \geq \gamma_{\rm F}$. These physical conditions are actually so powerful that we could have set the value of γ_{12} to lie between the narrow range of $\gamma_{\rm F}/\sqrt{2}$ and $\gamma_{\rm F}$ without carrying out much of the exercise done before. And the same conditions show that for $\gamma_{\rm F} \gg 1$, irrespective of the nature of the ambient medium, we would never have a solution which admits $\gamma_{12} << \gamma_{\rm F}$ (for instance $\gamma_{12} \approx \sqrt{\gamma_{\rm F}}$ discussed before). It is interesting to note that the value of γ_{12} should lie within such a narrow range. The physical constraints also imply that the maximum value of the bulk LF of the shocked material in the FB is $\sqrt{2}$, and thus the reverse shock in a GRB problem is bound to be non-relativsitic. From such considerations, we can approximately write

$$\frac{\gamma_{\rm F}}{\sqrt{2}} \leq \gamma_{12} \leq \gamma_{\rm F} \tag{25}$$

and

$$1 \leq \gamma_{24} \leq \sqrt{2} \tag{26}$$

Now we can go back to the work of Sari & Piran (1995) to see if the condition for occurrence of an ultrarelativistic reverse shock is valid or not. They have considered a case with $f \ll \gamma_{\rm F}^2$ (see eq. 5 of Sari & Piran 1995) for which $\gamma_{34} \gg 1$ (see our eq. 16). But, if we use our eq.(16), we immediately find that, for such a value of f, we would have $\gamma_{12} \ll \gamma_{\rm F}$, which is unphysical in view of the constraint (25). On the other hand, note that, eq. (22) and (23) are consistent with such physical constraints.

So, as long as we can assume that the radiative processes in both the forward-shocked fluid and the reverse-shocked fluid are sufficiently fast, we have $\tau \sim r_{\rm d}/c\gamma_{12}^2$. But since the reverse shock is actually, at best, mildly relativistic, particles are not expected to be accelerated to very high Lorentz factors within the limited observed duration of GRBs. And since for the radiative processes like synchrotron and inverse Compton, the time scales are inversely proportional to the LF of the particles, there is a possibility that the reverse-shocked fluid might significantly stretch the expected theoretical GRB time scales. How justified is this apprehension? The strength of the signal appearing from the two

regions (2) and (3) should depend on the ratio of the power dissipated in the two regions. And the latter should depend on the ratio of the amount of work done by S_1 on the ambient medium (1), and by S_2 on (4). Because the value of the fluid pressure is the same at S_1 and S_2 ($p_2 = p_3 = p$), the rate of compression or the rate of pdV work done by the two shocks is $\gamma_{S1} : \gamma_{S2} \approx \gamma_{\rm F} : 1 \approx \eta/2 : 1$. Since this rate is Lorentz invariant, and the expected value of $\eta_2 \sim 10^2 - 10^4$, we find that the amount of available power that goes into the compression of the FB is negligible. Accordingly, the rate of work done by the reverse-shock in any form, whether it is the heating of the FB, or producing enhanced magnetic field in it or producing low energy photons to facilitate inverse Compton boosted gamma-ray production in the region (2) is actually negligible.

Thus we come to a very important conclusion that we can practically ignore the reverse shock in studying the gross time scale of GRBs within the Meszaros and Rees framework! This above understanding enables us to write

$$\tau \approx \frac{a\ r_{\rm d}}{2c\eta^2} \tag{27}$$

where $8 < a < 16$. This value of τ is obviously one order higher than the usually quoted value of $\tau \approx r_{\rm d}/2c\eta^2$. By recalling the value of $r_{\rm d}$ from eqn. (21), we can rewrite

$$\tau \approx 0.1\ a\ E_{51}^{1/3}\ \eta_3^{-8/3}\ n_1^{-1/3} {\rm s} \tag{28}$$

For a given value of η, the above derived value of τ is at least one order large than similar values used in the literature (for instance see eqn. [5.2] of Mochkovitch et al, 1995)

4. DISCUSSION

We have been able to remove a basic qualitative deficiency in the theoretical description of GRBs in the Meszaros and Rees scenario, which has become the "standard model" for understanding the phenomenology of these events, i.e., whether $\tau \sim r_{\rm d}/2c\eta^2$ or $\tau \sim r_{\rm d}/2c\eta$.

Although, none of these expressions appeared to be quite correct our final result is obviously tilted in favour of the former relation which is, however, to be modified by a numerical factor $8 < a < 16$. We have also emphasized an important aspect in this problem: the minimum value of the ratio of the Lorentz invariant power liberated in the forward blast wave and the reverse shock goes as $\eta/2 \gg 1$. This point should considerably simplify the evaluation of the emitted spectrum in the problem. Although we have assumed a spherical FB, this discussion remains unchanged even for a case when the actual FB has a funnel type structure in the proper frame. To appreciate this subtle point, consider the creation of jet type FBs following a NS-NS collision event (Rees & Meszaros 1992). Suppose the $e^+ + e^-$ FB is due to collisions of ν and $\bar{\nu}$s emanating from the super hot and about-to-merge neutron stars. Then, despite the asymmetry in the binary geometry, $e^+ + e^-$ pairs would be produced over the full 4π solid angle, though there would be excess production along the symmetry axis because of larger value of collision angle along the same. Unless we are sure about the exact configuration of the neutrinosphere associated with each NS, or the configuration of the thick disk that may be formed as the result of merger, and also the temporal evolution of the whole pattern, it can only be a matter of conjecture as to how much excess production of pairs is achieved at a particular point with a given value of r and θ. The canonical values of $E \sim 10^{50}$ or 10^{51}erg is actually obtained by assuming a broad isotropic picture of the events, and one should actually consider the use of a fiducial value of E *per unit solid angle* for dealing the anisotropic cases. In case there is some excess production in a certain direction, *it does not mean that the pair flux produced in other directions somehow gets focussed in that direction* to result in the same canonical value of E assumed for the spherical 4π case. And the basic reason that there might be a funnel type geometry of the FB is not the somewhat excess production of pair in a given direction. The basic reason, instead, is that the mass flux of the debris of NS-NS merger event is assumed to be appreciably less in those preferred directions yielding a value of $\eta \gg 1$. If the merger event spews off a baryonic mass $\sim 10^{-3} M_\odot$, then even for an assumed (spherical) value of $E \sim 10^{51}$erg, in the isotropic case we would have $\eta \leq 1$, and there would be no GRB! It is primarily on this account that the assumption of relatively baryon free funnel along the

symmetry axis gives rise to jet-like FBs. It only means that there are other jet-like FBs too but they have unacceptably low value of η. Thus, in our view, once we keep a canonical value of E and tacitly assume an intrinsically spherical FB (though having various values of η in various directions), for a given observer, we must not additionally plug in the solid angle factor in associated quantities like $r_{\rm d}$ or τ, as have been done by Meszaros, Rees and Papathanassion (1994). To further appreciate this point, we can specifically consider two typical equations considered by them (2.4 and 2.16)

$$\tau \sim 1E_{51}^{1/3}\ \eta_3^{-8/3}\ n_1^{-1/3}\theta^{-2/3}\ \mathrm{s} \tag{29}$$

and the total burst fluence

$$S_0 = 10^{-6}\ E_{51}\ \theta^{-2}\ D_{28}^{-2}\ \mathrm{erg\ cm^{-2}} \tag{30}$$

where θ is the semi-angle of the funnel, S_0 is the total bolometric fluence of the burst occuring at a distance of $D = 10^{28}$cm. Note that, although, both *mathematically and physically* $\theta = 0$ *should correspond to absence of any burst, the two foregoing equations would suggest* $\tau \to \infty$, *and* $S_0 \to \infty$ *for a non-existent burst! This happens because the value of* E *was not scaled down in keeping with the value of* θ *in these equations virtually presuming that the canonical value for an isotropic FB energy somehow gets channelized along the narrow funnels.* Therefore, we reinterpret our basic result that in case the FB is conical with a solid angle $\Omega \sim \theta^2$ in its rest frame, then E_{51} is to be replaced by $\frac{\Omega}{4\pi}E_{51}$ to take care of the fractional energy channelization. It follows then that if we apply equations similar to (29) or (30), we, eventually get back the original eq.(28). In other words, the basic temporal properties of the FB (along a given direction) should be more or less unaltered even for an anisotropic case as long as we are able to define $\eta(\theta)$ in a meaningful way. It may be reminded here that although the generation of a relatively baryon free, high η jet-like FB is required for understanding the GRBs the understanding about their genesis is largely a matter of conjecture and is something like having "an artist's conception". This is so in view of the (i) uncertainty about the physical mechanism

triggering GRBs, (ii) even for an assumed mechanism like a NS-NS merger, the merger geometry is unknown and evolving faster (at the moment of maximum energy liberation) on timescales shorter than the observed GRB timescales, (iii) unknown effects like probable new general relativistic instabilities and the unknown nature and evolution of the coelesced object, (iv) the unknown extreme parameters like temperature, density and their profiles in the dynamic and unknown merger geometry like accretion disk, torus etc. etc., (v) the unknown microphysics at such extreme conditions like the equation of state and viscosity profiles of the dynamic merged object, and also the basic uncertainty about (vi) the relative importance of energy loss by gravitational radiation and neutrino emission.

The actual burst duration, even when we consider the emission of *hard X-rays and gamma rays only*, can obviously be longer than what is suggested by eq. (29) if the FB fails to radiate the available energy (50% at $r = r_{\rm d}$). In fact, when the burst fails to be radiative at $r \sim r_{\rm d}$, there may be a weak and prolonged burst corresponding to a larger value of r_γ. Also, if we redefine the burst duration as the one during which the FB radiates 75% of its energy we would have a value of τ larger by a factor of few ($\gamma_{\rm F} \rightarrow \gamma_{\rm F/2}$).

We remind the reader again that this whole discussion explored the question of duration of the GRB in the idealistic Meszaros & Rees framework which assumes that even in a sparse ISM ($n_1 \leq 1$), the hydrodynamic limit is achieved at any radius. Since the leading protons of the FB interact with the sparse ISM extremely weakly because the bacground ISM magnetic field is very weak, and we are not aware of any cooperative phenomemon by which these protons may collectively interact much more strongly by self generating strong magnetic fields on their way up, *it is quite uncertain* whether GRBs can be triggered in ordinary sparse ISM (Mitra 1996). Naturally, there may be no strong shock at all at $r = r_{\rm d}$ unless GRBs are hatched in special dense regions of ISM although subsequently the blastwave may propagate in the ordinary ISM and generate various low energy afterglow. The eventual mechanism of GRBs could be considerably different than what our present understanding admits and then part of the present discussion may be invalidated. For instance all the existing hydrodynamical models assume that *protons/ions accelerated in the shock can transfer their energy to the associated electrons sufficiently fast so that the*

radiative time scale of the shock is determined by the much shorter radiative time scale of the electrons. This assumption *need not be correct* and though it was pointed out privately by the present author to several other authors working on this problem this aspect has been glossed over for the sake of simplicity. We will discuss in a seperate paper under what condition the hydrodynamical description may be valid even at small values of r and how it may be possible to have a blast wave with a higher efficiency for gamma ray production (Mitra 1997).

It may be also possible that the fundamental mechamism for the GRBs is not an one shot collision process (involving compact objects) but, on the other hand, a process whereby energy is released erratically and in a jerky manner in the form of an "unsteady wind" over a duration of tens of seconds or even longer. In such a case the basic GRB duration will obviously be determined by the central source though afterglow time scales may be determined by hydrodynamic model discussed in this work.

Finally, let us point out that one can also seperately define $r_{\rm optical}$, $r_{\rm infrared}$, and $r_{\rm radio}$ in the context of the hydrodynamical model for the post GRB phase of the event provided one has a model for generation of respective radiations. After the main GRB phase, the value of γ would decrease rapidly and would approach unity, i.e, the blast wave will become Newtonian like a SNR. At each stage the afterglow will be characterized by a time scale $\tau_{\rm radiation} \sim \frac{r_{\rm radiation}}{\gamma^2 c}$ with no simple and general relationship between $\tau_{\rm radiation}$ and η. Such afterglow time scales can obviously be arbitrarily long and exceed even hundreds of years if the GRB event occurs in the Local Group. Thus, if we are fortuitous, we may be able to identify some of the presenly known SNRs which have no signature of harbouring a compact object at their centers as Gamma Ray Burst Remnants (GRBR).

Acknowledgement : The author is thankful to the anonymous referee for several constructive suggestions which greatly improved the presentation of this work.

REFERENCES

Blandford, R.D. & McKee, C.F. 1976, Phys. Fluids, 19, 1130

Cavallo, G. & Rees, M.J. 1978, MNRAS, 183, 359

Eichler, D, Livio, M., Piran, T., & Schramm, D. 1989, Nature, 340, 126

Goodman, J. 1986, ApJ, 308, L47

Katz, J.I. 1994, ApJ, 422, 248

Meszaros, P. & Rees, M.J. 1993, ApJ, 405, 278

Meszaros, P., Laguna, P., & Rees, M.J. 1993, ApJ, 215, 181

Meszaros, P., Rees, M.J., & Papathanssiou, H. 1994, ApJ, 432, 181

Mitra, A. 1996, A&A, 313, L9

Mitra, A. 1997 (in preparation)

Mochkovitch, R. et al. 1995, A&A, 293, 803

Paczynski, B. 1986, ApJ, 308, L47

Piran, T., Shemi, A., & Narayan, R. 1993, MNRAS, 263, 861

Piran, T. 1994 in Gamma-Ray Bursts, Second Workshop, ed. G.J. Fishman, J.J. Brainerd, & K. Hurley (New York: AIP)

Rees, M.J. 1966, Nature, 211, 468

Rees, M.J. & Meszaros, P. 1992, MNRAS, 258, 41P

Rees, M.J. & Meszaros, P. 1944, ApJ, 430, L93

Sari, R. & Piran, T. 1995, ApJ, 455, L146

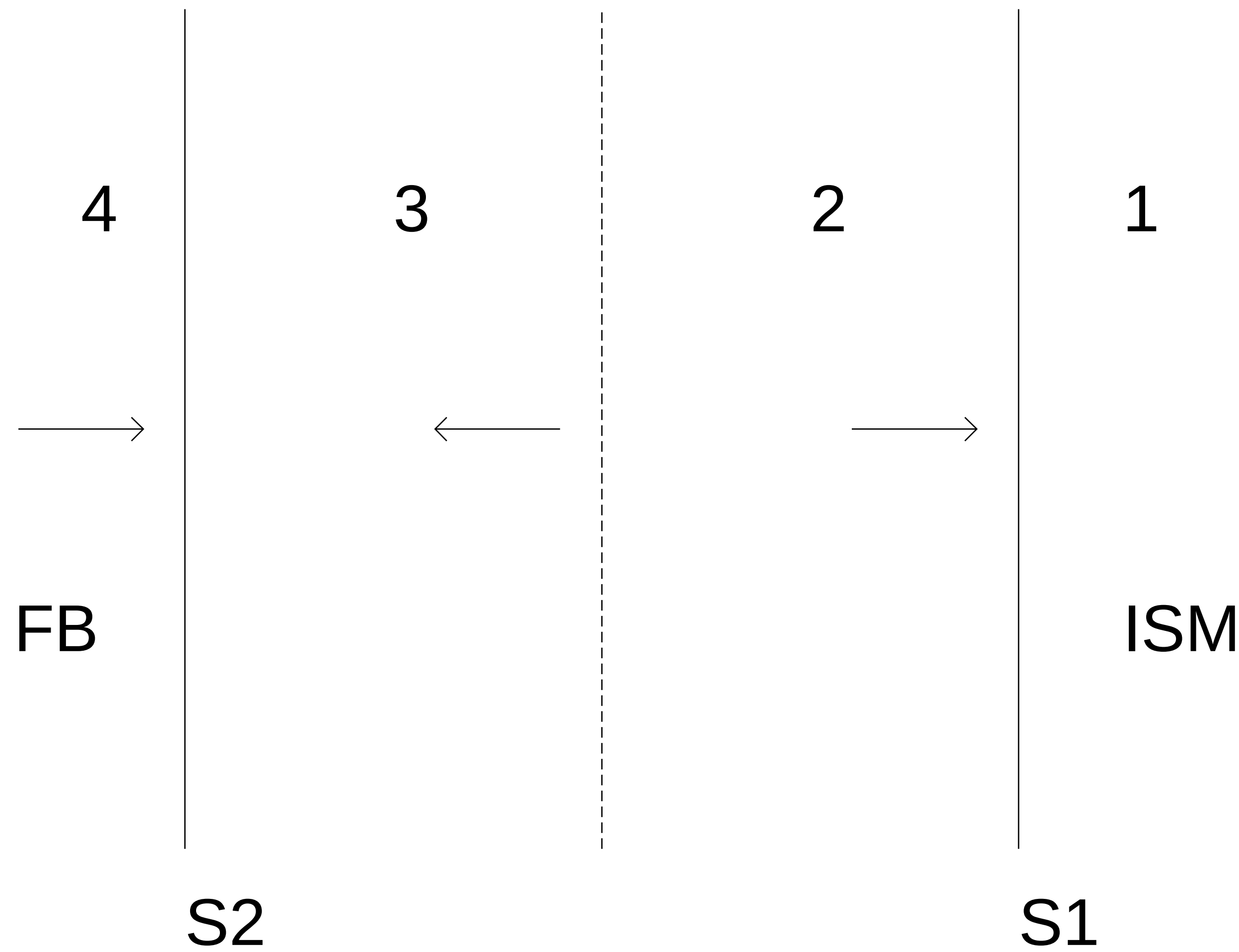
Shemi, A. & Piran, T. 1990, ApJ, 365, L55

Taub, A.H. 1949., Phys. Rev., 74, 328

This manuscript was prepared with the AAS LaTeX macros v4.0.

Figure Caption

Figure 1: Sketch of the FB-shock configurations, for details, see text.


4
3
2
1
FB
ISM
S2
S1